Learning about flavour structure from $\tau \to e_\beta \gamma$ and $\mu \to e\gamma$?

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Abstract

Current and upcoming experiments should improve the sensitivity to $e_\alpha \to e_\beta \gamma$ decays by an order of magnitude. This paper assumes that one of the $\tau \to e_\beta \gamma$ decays is observed, and explores the structure and consequences of the required new flavoured couplings. In simple models (a low-scale seesaw, leptoquarks) it is shown that the dipole vertex function is proportional to a product of flavoured matrices from the Lagrangian (a "Jarlskog-like" invariant), provided that the loop particles are weakly coupled to the Higgs. Secondly, if the dipole vertex function has a hierarchical structure, this can imply that only some of the $\tau \to e_\beta \gamma$ modes can be observed, due to the "approximate zero" implied by the bound on $\mu \to e\gamma$. The assumptions underlying this potential test of a hierarchical structure are discussed.

1 Introduction

Measuring $e_\alpha \to e_\beta \gamma$ would indicate New Physics near the TeV scale. Various studies of TeV New Physics models anticipate correlations in the Lepton Flavour Violating rates for $e_\alpha \to e_\beta \gamma$ and other processes. These predictions depend on the particle content and masses, the flavoured couplings of the model, and on the parameter space scan. This paper focuses on the flavour structure of New Physics: what can $e_\alpha \to e_\beta \gamma$ tell us about patterns of lepton flavour violation, with limited knowledge of the new particle content?

After electroweak symmetry breaking, the dipole operator which induces $e_\alpha \to e_\beta \gamma$, can be written,

$$\frac{e}{16\pi^2}\left([X_L]_{\beta\alpha}\overline{e_\beta}\sigma^{\mu\nu}P_L e_\alpha F_{\mu\nu} + [X_R]_{\beta\alpha}\overline{e_\beta}\sigma^{\mu\nu}P_R e_\alpha F_{\mu\nu}\right) \quad . \tag{1}$$

where $\sigma^{\mu\nu} = i[\gamma^\mu, \gamma^\nu]/2$. The dipole "vertex function"[†] $X_{L,R}$ can be expressed as a dimensionful unflavoured constant, multiplied by a dimensionless "flavour tensor". A first question, is when and how does the $e_\alpha \to e_\beta \gamma$ flavour tensor reflect the flavoured couplings of the Lagrangian ("spurions")? We study, at one loop in two models — the type I seesaw and leptoquarks — when the $e_\alpha \to e_\beta \gamma$ flavour tensor can be constructed, like a Jarlskog invariant [1], by contracting flavoured tensors present in the Lagrangian.

Experimental searches for Lepton Flavour Violating (LFV) radiative decays impose significant bounds on the flavour changing dipole vertex function. The MEG [2] experiment constrains $BR(\mu \to e\gamma) < 2.4 \times 10^{-12}$ and plans to reach a sensitivity of 10^{-13} in the coming years. Babar and Belle have searched for $\tau \to e_\beta \gamma$ decays, and impose $BR(\tau \to \mu\gamma) < 4.4 \times 10^{-8}$[3, 4] and $BR(\tau \to e\gamma) < 3.3 \times 10^{-8}$ [3] (see table 1). Super-B factories expect a sensitivity $BR \sim 10^{-9}$ [5, 6, 7]. A second aim of this paper, is to explore whether Super-B factories, combined with MEG, can test the flavour structure of the dipole vertex function. So we make the non-trivial assumption that one of the $\tau \to e_\beta \gamma$ rates is within the reach of the Super-B factories. This is a phenomenologically interesting scenario to envisage, because observing one $\tau \to e_\beta \gamma$ rate gives two pieces of information, the second one being an "approximate zero" in the dipole flavour tensor, imposed by the $\mu \to e\gamma$ bound. We arge that, if the flavour tensor is hierarchical[‡] and certain other assumptions are satisfied, then one of the remaining $\tau \to e_\beta \gamma$ rates is suppressed below Super-B sensibilities. The assumptions required for this potential "test" of the hierarchical structure are discussed.

Section 2 gives the $e_\alpha \to e_\beta \gamma$ branching ratios, reviews various properties of the dipole vertex function $\mathbf{X}_{L,R}$, and presents one-loop formulae for it. At the end of the section, is discussed the relation of $\mathbf{X}_{L,R}$ to the coefficient of the dimension six, electroweak gauge invariant, dipole operator. In the models considered in sections 3, which are a TeV-scale seesaw, and various scalar leptoquarks, the vertex function $\mathbf{X}_{L,R}$ turns out to be proportional to an

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[†]The $\mathbf{X}_{L,R}$ are arbitrary functions of coupling constants and masses from the Standard Model and beyond. They are refered to as vertex functions, to distinguish them from coefficients of operators in an expansion in $1/\Lambda_{NP}$, where Λ_{NP} is a New Physics scale.

[‡]meaning that it is dominated by its largest eigenvalue, analogously to $[Y_u Y_u^\dagger]_{ij} \simeq y_t^2 V_{ti}^* V_{tj}$

$\widetilde{BR}$	current bound	future
$\mu \to e\gamma$	$2.4 \times 10^{-12}[2]$	$\sim 10^{-13}, (\text{MEG } [2])$
$\tau \to \mu\gamma$	$2.5 \times 10^{-7}[3,\ 4]$	$\sim 10^{-8}, (\text{super-B factories}[5])$
$\tau \to e\gamma$	$1.9 \times 10^{-7}[3]$	$\sim 10^{-8}, (\text{super-B factories}[5])$

Table 1: Current bounds and hoped-for sensitivities to lepton flavour violating branching ratios, normalised to leptonic weak decays, as in eqn (2).

"invariant", when it is the coefficient of this dimension six operator. The definition of what we mean by "invariant", can be found at the beginning of section 3. Section 4 reviews the argument that a hierarchical structure can be tested. Finally, the prospects of obtaining invariants and testing a hierarchy are discussed in section 5. A summary of useful loop results from a paper by Lavoura[8] appears in the Appendix.

2 Notation and review

For reviews of the expectations and prospects for lepton flavour violating processes, see *e.g.* [6, 7, 9, 10]. The theoretical predictions have been widely studied in popular models such as Left-Right models [11], multiple Higgs[12], supersymmetry [13, 14, 15, 16], Little Higgs models [17, 18] and extra dimensions [19, 20, 21]. The expectations of an A4 flavour symmetry, implemented in dimension six lepton flavour violating operators (so a New Physics model is not required), were explored in [15]. A more model-independent approach [13, 22], uses a "penguin-box" expansion [23], to classify New Physics models according to the dominant effective interactions which they induce among (three or four) SM particles. The various possibilities predict correlations [13, 16, 22, 24, 25] among observables (such as $(g-2)_\mu$ and $\mu \to e\gamma$[26], $\mu \to \bar{e}e\bar{e}$ and $\mu \to e\gamma$[27], or $\mu \to e\gamma$ and μe conversion[28]).

This paper studies flavour structure in the three $e_\alpha \to e_\beta\gamma$ decays, which are induced by the three-particle dipole vertex given in eqn (1). Electric and magnetic dipole moments are neglected because they are flavour diagonal (see the expansion (3)), and other observables are usually neglected because they are induced by other operators.

Lepton flavour violating radiative decays, $e_\alpha \to e_\beta\gamma$, proceed via the operator (1) at a rate given by

$$\widetilde{BR}(e_\alpha \to e_\beta\ \gamma) \equiv \frac{\Gamma(e_\alpha \to e_\beta\ \gamma)}{\Gamma(e_\alpha \to e_\beta \nu_\alpha \bar{\nu}_\beta)} = \frac{\alpha_{em}m_\alpha^3}{256\pi^4}(|X_{L\beta\alpha}|^2 + |X_{R\beta\alpha}|^2)\frac{192\pi^3}{G_F^2 m_\alpha^5} \quad , \tag{2}$$

where $\alpha, \beta \in \{e, \mu, \tau\}$, and the whole paper uses the charged lepton mass eigenstate basis. Notice that in table 1, as in the remainder of this paper, the $\tau \to e_\beta\gamma$ "branching ratios with tilde" are defined with respect to the leptonic τ decay rate to facilitate comparaison with $\mu \to e\gamma$. Table 1 lists the current bounds, and hoped for sensitivities of running or planned experiments.

The dipole vertex functions of eqn (1) satisfy $\mathbf{X}_R^\dagger = \mathbf{X}_L$: $[\mathbf{X}_R]_{e\mu}$ induces $\mu_R \to e_L\gamma$, $[\mathbf{X}_L]_{e\mu}$ induces $\mu_L \to e_R\gamma$, $[\mathbf{X}_R]_{\mu e}$ induces $(\mu_L)^+ \to (e_R)^+\gamma$, and $[\mathbf{X}_L]_{\mu e}$ induces $(\mu_R)^+ \to (e_L)^+\gamma$. Since we are interested in $e_\alpha \to e_\beta\gamma$ rates (and not CP violation), we only need to consider the $\alpha > \beta$ components (lower triangle) of $[\mathbf{X}_R]_{\beta\alpha}$ and $[\mathbf{X}_L]_{\beta\alpha}$, which are independent. The dipole vertex functions can be written as :

$$\mathbf{X}_R = v\,(c_R\mathbf{Y^e} + \mathbf{S}_L\mathbf{Y^e} + \mathbf{T}_R\,)\frac{4G_F}{\sqrt{2}} \tag{3}$$

$$\mathbf{X}_L = v\left(c_L\mathbf{Y^{e\dagger}} + \mathbf{S}_R\mathbf{Y^{e\dagger}} + \mathbf{T}_L\right)\frac{4G_F}{\sqrt{2}} \quad ,$$

where $v = 174$ GeV is the Higgs vev. The normalisation by $2\sqrt{2}G_F$ is concrete, however, the masses of the new particles which mediate $e_\alpha \to e_\beta\gamma$ will be assumed to exceed the weak scale v. $\mathbf{Y}_e$ is the (diagonal) charged lepton Yukawa matrix, c_L and c_R are unknown, unflavoured constants, and $\mathbf{S}_L$, $\mathbf{S}_R$ and $\mathbf{T}_{L,R}$ are new dimensionless matrices in flavour space, which we will refer to as "flavour tensors". Section 3 studies when they can be constructed by multiplying together flavoured matrices that one finds in the Lagrangian. $\mathbf{S}_L$ and $\mathbf{S}_R$ are hermitian, respectively induced by New Physics which couples to doublet leptons, or to singlet leptons. $\mathbf{T}_R = \mathbf{T}_L^\dagger$ is an arbitrary matrix (as was $\mathbf{X}$), $\mathbf{T}_R$ is defined like the Yukawas to have doublet-singlet index order, induced by New Physics interacting with doublet and singlet leptons. The $\mathbf{X}_L, \mathbf{X}_R$ are labelled by the chirality of the incoming charged lepton; the $\mathbf{S}_{L,R}$ have the opposite chiral label, because the incoming chirality is flipped by a charged lepton Yukawa coupling.
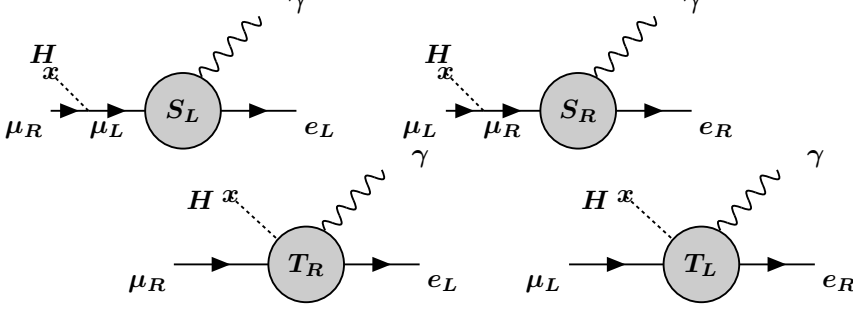
Figure 1: The first blob corresponds to $[\mathbf{S}_L]_{e\mu}$, the second to $[\mathbf{S}_R]_{e\mu}$. On the second row, the blobs correspond to $[\mathbf{T}_R]_{e\mu}$ and $[\mathbf{T}_L]_{e\mu}$.

The decomposition of eqn (3) corresponds to flavour diagonal + flavour changing new physics that is not chirality-flip (the $\mathbf{S}_{L,R}$ terms therefore contains Y_e) + flavour changing chirality flip new physics represented by $\mathbf{T}_{L,R}$. The first term $\propto \mathbf{Y}_e$, whose coefficient can describe the magnetic and electric dipole moments, can not induce the flavour off-diagonal $e_\alpha \to e_\beta \gamma$. This term is nonetheless included, so that the diagonal elements of our new flavour tensors are not required to parametrise flavour diagonal new physics. The lower triangles of $\mathbf{S}_{L,R}$, $\mathbf{T}_L$ and $\mathbf{T}_R$ correspond respectively to the four diagrams of figure 1. We have neglected the possible $\mathbf{Y}_e^\dagger \mathbf{S}_L$ contribution to $\mathbf{X}_L$, and the $\mathbf{Y}_e \mathbf{S}_R$ contribution to $\mathbf{X}_R$, because they are proportional to the outgoing charged lepton mass m_β.

In the presence of either the $\mathbf{S}_{L,R}$ or $\mathbf{T}$ flavour tensors [§], the ratios of branching ratios of eqn (2) become (recall that $4G_F/\sqrt{2} = 1/v^2$, for $v = 174$ GeV):

$$\widetilde{BR}(l_\alpha \to l_\beta\ \gamma) = \frac{6\alpha}{\pi}\left(|S_{L\beta\alpha}|^2 + |S_{R\beta\alpha}|^2\right)$$
$$\sim 1.4 \times 10^{-2}\left(|S_{L\beta\alpha}|^2 + |S_{R\beta\alpha}|^2\right) \tag{4}$$

$$\widetilde{BR}(l_\alpha \to l_\beta\ \gamma) \sim \frac{140 m_\tau^2}{m_\alpha^2}\left(|T_{L\beta\alpha}|^2 + |T_{R\beta\alpha}|^2\right) \quad . \tag{5}$$

If the the dimensionless couplings in $T_{X\beta\alpha}$ and $S_{X\beta\alpha}$ are taken $\lesssim 1$, such that $S_{X\beta\alpha} \sim v^2/\Lambda_{NPFC}^2$ and $T_{X\beta\alpha} \sim v^2/\Lambda_{NPFC}^2$, then Super-B factories could probe to a new flavour changing scale $\Lambda_{NPFC} \lesssim 10$ TeV for $\mathbf{S}_{L,R}$ and $\Lambda_{NPFC} \lesssim 60$ TeV for $\mathbf{T}$.

Convenient formulae for the $[X_X]_{\beta\alpha}$ vertex function, generated in one-loop diagrams involving a boson and a fermion f (see $e.g.$ figures 2 and 3), are presented in the paper of Lavoura [8]. For a scalar S with Yukawa-type interactions

$$\mathcal{L}_{(Lavoura)} = S\overline{f}(\lambda_{L\sigma}P_L + \lambda_{R\sigma}P_R)e_\sigma + h.c. \tag{6}$$

where e_σ is a charged lepton of flavour σ, in four component notation, Lavoura obtains

$$\frac{[S_R]_{\beta\alpha}}{v^2} = -\frac{\rho_{\beta\alpha}}{2m_S^2}\left[Q_f k(t) + Q_S \overline{k}(t)\right] \tag{7}$$

$$\frac{[S_L]_{\beta\alpha}}{v^2} = -\frac{\omega_{\beta\alpha}}{2m_S^2}\left[Q_f k(t) + Q_S \overline{k}(t)\right] \tag{8}$$

$$\frac{[T_R]_{\beta\alpha}}{v^2} = -\frac{m_f \xi_{\beta\alpha}}{2m_S^2 v}\left[Q_f k_f(t) + Q_S \overline{k_f}(t)\right] \tag{9}$$

$$\frac{[T_L]_{\beta\alpha}}{v^2} = -\frac{m_f \zeta_{\beta\alpha}}{2m_S^2 v}\left[Q_f k_f(t) + Q_S \overline{k_f}(t)\right] \tag{10}$$

where the electric charges satisfy $Q_S - Q_f = 1$, the dimensionless functions $k, \overline{k}, k_f$ and $\overline{k_f}$ of the mass ratio $t = m_f^2/m_S^2$ are given in the Appendix, and

$$\omega_{\beta\alpha} = \lambda_{L\beta}^* \lambda_{L\alpha} \quad , \quad \rho_{\beta\alpha} = \lambda_{R\beta}^* \lambda_{R\alpha} \quad , \quad \xi_{\beta\alpha} = \lambda_{L\beta}^* \lambda_{R\alpha} \quad , \quad \zeta_{\beta\alpha} = \lambda_{R\beta}^* \lambda_{L\alpha} \quad . \tag{11}$$

[§]if both are present, there are ST interference terms

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Lavoura also gives formulae for a vector and fermion loop. Suppose that W bosons interact with $e_{L\sigma}$ and a neutral fermion f via the Lagrangian

$$\mathcal{L}_{vec} = A'_{L\sigma} W_\rho \overline{f} \gamma^\rho e_{L\sigma} + h.c. + \delta\mathcal{L}_{F'H} \qquad (12)$$

where $A'_{L\sigma} = g/\sqrt{2}\times$mixing angles, and $\delta\mathcal{L}_{F'H}$ is the additional interactions, in Feynman 'tHooft gauge, of the Higgs goldstone components. Then Lavoura gives

$$\frac{[S_R]_{\beta\alpha}}{v^2} = -\frac{\tilde{\rho}_{\beta\alpha}}{2m_W^2} Q_W \overline{y}(t) \quad , \quad \text{where} \quad \tilde{\rho}_{\beta\alpha} = A'^*_{L\beta} A'_{L\alpha} \qquad (13)$$

$\overline{y}$ is given in the Appendix, and $[S_L]_{\beta\alpha} = [T_R]_{\beta\alpha} = [T_L]_{\beta\alpha} = 0$.

In the $SU(2)\times U(1)$ invariant Lagrangian, there is a dimension six operator $O^{e\gamma}$, which contributes to the photon dipole operator, and which is a linear combination of hypercharge and weak SU(2) operators:

$$O^{e\gamma}_{\beta\alpha} \equiv (\overline{\ell}_\beta H)\sigma^{\mu\nu} P_R e_\alpha F_{\mu\nu} = \cos\theta_W (\overline{\ell}_\beta H)\sigma^{\mu\nu} P_R e_\alpha B_{\mu\nu} - \sin\theta_W (\overline{\ell}_\beta \tau^3 H)\sigma^{\mu\nu} P_R e_\alpha W^3_{\mu\nu} \qquad (14)$$

where ℓ is an SU(2) doublet, $\sigma^{\mu\nu} = i[\gamma^\mu, \gamma^\nu]/2$, and $\{\tau^i\}$ are the Pauli matrices. If the operator of eqn (14) appears in the Lagrangian normalised as $\mathcal{L} \supset e2\sqrt{2}G_F \mathbf{C}^{(6)}_{\beta\alpha} O^{e\gamma}_{\beta\alpha} + h.c.$, then $\mathbf{C}^{(6)}_{\beta\alpha}$ contributes to $[\mathbf{X}_R]_{\beta\alpha}$ (see eqn (3)). Notice that there could also be higher dimensional dipole operators, such as

$$O^{e\gamma H^2}_{\beta\alpha} \equiv (H^\dagger H)(\overline{\ell}_\beta H)\sigma^{\mu\nu} P_R e_\alpha F_{\mu\nu} \qquad (15)$$

with additional Higgs legs and suppressed by more powers of the flavour changing New Physics scale Λ_{NPFC}. Therefore, $\mathbf{C}^{(6)}$ corresponds to the $\mathcal{O}(1/\Lambda^2_{NPFC})$ terms in $\mathbf{X}_R$, and the coefficients of $O^{e\gamma H^2}$ include $\mathcal{O}(m^2_{SM}/\Lambda^4_{NPFC})$ contributions[¶] (where m_{SM} is a SM mass). In this paper, we assume that there is a New Physics contribution to $\mathbf{C}^{(6)}$ and we will be interested in identifying cases where $\mathbf{C}^{(6)} \simeq \mathbf{X}_R$.

3 Invariants

For the purposes of this paper, invariants are tensors in flavour space, often a scalar or a matrix, obtained by multiplying flavoured matrices from the Lagrangian ("spurions", in Minimal Flavour Violation [29] langauge). They are interesting for Beyond the Standard Model physics, because, if a sufficient number of invariants are measured, the flavoured matrices of the Lagrangian can be "reconstructed" by simple matrix multiplication. Like the original invariant of Jarlskog [1], these invariants are also an elegant way to avoid dependance on unphysical or unknown basis choices in the Lagrangian. So the invariants in this paper are allowed to have flavour indices, but these must be in the mass eigenstate basis of known particles.

It is clear that that the formulae (7) - (10) and (13) have the makings of "Jarlskog-like invariants", at least in the case where the k-functions can be approximated by a single term. For instance, if $m_f \ll m_S$, then $\mathbf{S}_{L,R} \sim \lambda \mathbf{m}^{-2}_S \lambda^\dagger$. This would be a coefficient of the dimension six operator given in eqn (14). The remaining terms from the k functions, $\propto (m_f/m_S)^n$ for $n \geq 1$, would give coefficients for dimension > 6 operators which give more suppressed contributions to the dipole vertex functions. The first aim of this paper, is to identify when the flavour tensors $\mathbf{S}_{L,R}$, $\mathbf{T}_L$ and $\mathbf{T}_R$ are invariants. Various questions come to mind about this connection: what happens to logarithms? What if both particles in the loop are flavoured? What happens when there are several new particles with slightly different masses? Whether the vertex functions are invariants is essentially a "top-down" question. However, it is phenomenologically interesting, because we would like to reconstruct the fundamental Lagrangian from observables, so it is useful to know when the connection is simple.

3.1 The non-supersymmetric Seesaw

The type I seesaw [30], (without supersymmetry), is simple model in which to study the invariant question, because it only contributes to $\mathbf{S}_L$, and because only one of the particles exchanged in the loop (the neutral fermion) carries a generation index. The singlet neutrino masses are assumed to be at the TeV-scale[31], as can for instance arise in the "inverse seesaw" [32]. Some recent constraints on such models can be found in [33, 34]. This mass scale is chosen so as to obtain detectable $e_\alpha \to e_\beta \gamma$ rates [35, 36], while maintaining the new mass scale above the Higgs vev v. (The seesaw behaviour works for singlet masses $\ll$ TeV [37], but our discussion will not apply.)

[¶]The coefficient of $O^{e\gamma H^2}$ would also include the contributions $\propto v^2$ to the mass^2 of the new particles. However, if we propagate mass eigenstate new particles, such contributions would remain resummed on the denominator.

Section 3.1.1 approximately diagonalises the neutral mass matrix, and checks that the known $e_\alpha \to e_\beta\gamma$ amplitudes are reproduced [||] using the formulae of [8] for the neutral fermion and W loop diagram. Section 3.1.2 studies the conditions under which the amplitudes are proportional to invariants.

3.1.1 The model

Consider adding r heavy $(M_I \gtrsim \text{TeV})$ singlet fermions N, with a majorana mass matrix M, to the three generation SM. In a simple inverse seesaw model, there would be $r = 6$ singlets, arranged in three pairs of opposite-sign, but almost-equal magnitude, masses. The leptonic Lagrangian, in the mass basis of charged leptons and heavy singlets, will be:

$$\mathcal{L} = \lambda_{\alpha I}\bar{\ell}_\alpha H_u^* N_I - \frac{M_I}{2}N_I N_I - y_\alpha \bar{\ell}_\alpha H_d^* e_\alpha + h.c. \tag{16}$$

where $\bar{\ell}H_u^* = \bar{e}(H_u^+)^* - \bar{\nu}(H_u^0)^*$, and two Higgs doublets are allowed for. The neutral mass terms will be

$$\mathcal{L}_{mass} \to -\lambda_{\alpha I}\overline{\nu_\alpha}\langle H_u^{0*}\rangle N_I - \frac{M_I}{2}N_I N_I + h.c.$$

and the resulting $(3 + r) \times (3 + r)$ majorana mass matrix can be diagonalised with a unitary matrix X:

$$X\begin{bmatrix} 0 & \lambda v_u \\ \lambda^T v_u & M \end{bmatrix}X^T = \begin{bmatrix} D_m & 0 \\ 0 & D_M \end{bmatrix} \quad \text{where} \quad D_m \equiv -v_u^2 U^\dagger \lambda M^{-1}\lambda^T U^* \tag{17}$$

where $D_M = \text{diag}\{M_1, ..M_r\}$, and we suppose that it is sufficient to obtain X to order $1/M^2$, because $O^{e\gamma}$ of eqn (14) is a dimension six operator. Assuming that $\det[M] \neq 0$, because all the singlets are heavy, and taking U to be the usual leptonic mixing matrix, gives

$$X = \begin{bmatrix} U^\dagger & 0 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 1 - \delta_1 & -\lambda M^{-1} \\ M^{-1*}\lambda^\dagger & 1 - \delta_2 \end{bmatrix} \tag{18}$$

where the δ_i ensure that X is unitary at $O(1/M^2)$:

$$\delta_1 = \frac{v_u^2}{2}\lambda[M^\dagger M]^{-1}\lambda^\dagger \quad , \quad \delta_2 = \frac{v_u^2}{2}[M^\dagger]^{-1}\lambda^\dagger\lambda M^{-1} \tag{19}$$

A flavour eigenstate neutrino, participating in a vertex with the W and a charged lepton, can therefore be expanded on light (ν_i) and heavy (n_J) four-component majorana mass eigenstates as

$$\nu_\alpha = \sum_i \left(U_{\alpha i} - \frac{v_u^2}{2}[\lambda[M^\dagger M]^{-1}\lambda^\dagger U]_{\alpha i}\right)\nu_i + \sum_J \frac{v_u\lambda_{\alpha J}}{M_J}n_J \tag{20}$$

3.1.2 To get invariants?

The contribution to the dipole flavour tensors is obtained by summing the diagrams where a W boson and a light or heavy neutral fermion is exchanged. The relevant diagrams are given in figure 2. We neglect the light neutrino mass[2] contribution because it is parametrically of $\mathcal{O}(\lambda^4)$, to be compared to the $\mathcal{O}(\lambda^2)$ contributions from heavy neutral exchange and from the "non-unitarity" of the light neutrals.

Combining eqn (13) with eqn (20) gives

$$\frac{[S_R]_{\beta\alpha}}{v^2} = -\frac{g^2 v_u^2}{4m_W^2}[\lambda[M^\dagger M]^{-1}\lambda^\dagger]_{\beta\alpha}\left(-\overline{y}_1(0) + \overline{y}_1(M_J^2/m_W^2)\right)$$

$$= -\frac{s_\beta^2}{4}[\lambda[M^\dagger M]^{-1}\lambda^\dagger]_{\beta\alpha} + ... \qquad \text{heavy + light in loop} \tag{21}$$

where $v_u/v \equiv s_\beta$ and $\overline{y}$ is given in eqn (49), and only the $\mathcal{O}(1/M^2)$ terms were retained. This is consistent with calculating the diagonalisation matrix X to $\mathcal{O}(1/M^2)$.

[||] Despite comments in [34], the correct amplitudes can also be obtained by first matching the unbroken gauge theory at the heavy scale M onto the dimension six "kinetic" operator $\bar{\ell}H \overleftrightarrow{\not{D}} H^*\ell$ of [20] and the dipole operator of eqn (14), and then matching at m_W, in the broken theory, onto the dipole, including the kinetic operator in the loop. At the high-scale matching, it seems that the not-1PI diagrams with a Higgs insertion on the external leg should be included.
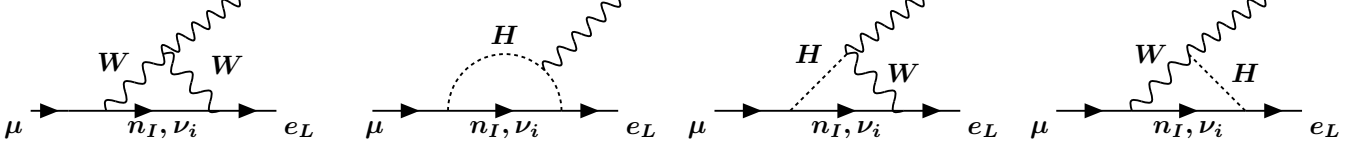
Figure 2: One-loop diagrams that contribute to the dipole vertex function in the seesaw model with broken electroweak symmetry. H is the goldstone. A Higgs leg, which is not drawn, can attach to the μ or n_I line.

As hoped for, equation (21) is an invariant constructed from flavoured matrices from the Lagrangian. This is the case despite electroweak symmetry breaking and the separate contributions from the scales M and m_W. However, there are $\mathcal{O}(v^2/M^4)$ contributions from higher order terms in the expansion of $\overline{y}$. From eqn (49), one sees that heavy and light neutral exchange will give contributions with coefficients proportional to

$$v^2[\lambda[M^\dagger M M^\dagger M]^{-1}\lambda^\dagger] \quad ... \quad , \quad [\lambda M^{-1}\lambda^T\lambda^* M^{-1*}\lambda^\dagger] \; , \; v^2[\lambda[M^\dagger M]^{-1}\lambda^\dagger\lambda M^{-1}\lambda^T\lambda^* M^{-1*}\lambda^\dagger] \quad ... \tag{22}$$

Since the eigenvalues and eigenvectors of the neutral mass matrix were only obtained to $\mathcal{O}(1/M^2)$, the numerical factors multiplying the $\mathcal{O}(v^2/M^4)$ terms are unknown. The coefficients are still invariants, which seems reasonable when the flavoured particles in the loop all have masses beyond current experimental reach. Fortunately, there are no logarithms at dimension six.

3.2 Scalar leptoquarks

The second toy model is various scalar leptoquarks. For updated low energy bounds, and references to earlier works, see *e.g.* [38]. The current collider lower bound on the mass of a leptoquark decaying to first generation leptons and quarks is 660 GeV [45], based on 1 fb^{-1} of data. See [44] for earlier collider mass bounds.

The scalar leptoquark contribution to the dipole vertex function is more complicated than the seesaw model, because they can contribute to $\mathbf{S}_L, \mathbf{S}_R$, or $\mathbf{T}$ (because they can interact with both singlet and doublet charged leptons), and in that both they and the quark in the loop can be flavoured.

Subsection 3.2.1 gives the Lagrangian for baryon and lepton number conserving scalar leptoquarks, and the contributions to $e_\alpha \to e_\beta\gamma$ of two SU(2) singlet leptoquarks upon which we focus. These are representative of the possibilities (see table 2), because the S_0 leptoquark can interact with singlet and doublet fermions. Then subsection 3.2.2 discusses the prospects for obtaining invariants.

3.2.1 The models

Consider scalar leptoquarks, with renormalisable B and L conserving interactions, which can be SU(2) singlets, doublets or triplets. In the notation of Buchmuller,Rückl and Wyler[39], these can be added to the SM Lagrangian as:

$$\begin{aligned}
\mathcal{L}_{LQ} &= S_0(\lambda_{\mathbf{LS_0}}\overline{\ell}i\tau_2 q^c + \lambda_{\mathbf{RS_0}}\overline{e}u^c) + \tilde{S}_0\tilde{\lambda}_{\mathbf{R\tilde{S}_0}}\overline{e}d^c \\
&\quad + (\lambda_{\mathbf{LS_2}}\overline{\ell}u + \lambda_{\mathbf{RS_2}}\overline{e}q[i\tau_2])S_2 + \tilde{\lambda}_{\mathbf{L\tilde{S}_2}}\overline{\ell}d\tilde{S}_2 \\
&\quad + (\lambda_{\mathbf{LS_3}}\overline{\ell}i\tau_2\vec{\tau}q^c)\vec{S_3} + h.c.
\end{aligned} \tag{23}$$

where the λs are 3×3 matrices with index order lepton-quark, and $\{\tau_i\}$ are Pauli matrices, so $i\tau_2$ provides the anti-symmetric SU(2) contraction. In this Lagrangian, the leptoquark leaves the vertex into which enter the leptons. This is converse to Lavoura conventions, where scalar and lepton both enter, and the internal fermion leaves. Comparing eqn (23) with eqn (6) gives the parameters listed in table 2. We will also allow the possibility of three generations [40] of leptoquarks, in whiich case $I : 1..3$, and λ_{lq}^I is a three-index tensor.

The SU(2) **singlet leptoquark** $\tilde{S}_0$, which couples to $\overline{d^c}e$, contributes to $\mathbf{S}_R$ via the two diagrams summarised by the left diagram in figure 3 (the photon can attach to S or f), where the internal fermion f can be a singlet d^c, s^c or b^c quark. For a $\widetilde{S_0}$ leptoquark, eqn (7) implies that $\mathbf{S}_L = \mathbf{T}_L = \mathbf{T}_R = 0$, and

$$\frac{[\mathbf{S}_R]_{\beta\alpha}}{v^2} = -\sum_f\left[\frac{\rho_{\beta\alpha}}{2M_{\tilde{S}}^2}\left(k(x) + 4\overline{k}(x)\right)\right] \simeq -\frac{1}{4M_{\tilde{S}}^2}[\tilde{\lambda}_{R\tilde{S}_0}\tilde{\lambda}_{R\tilde{S}_0}^\dagger]_{\beta\alpha} \quad , \quad \text{leptoquark without generations} \tag{24}$$

leptoquark	Q_S	Q_f (f)	ω	ρ	ξ	ζ
$\tilde{S}_0^\dagger$	4/3	1/3 (d^c)	0	$[\tilde{\lambda}_{R\tilde{S}_0}]_{\beta f}[\tilde{\lambda}_{R\tilde{S}_0}]^*_{\alpha f}$	0	0
$S_0^\dagger$	1/3	$-2/3$ (u^c)	$[\lambda_{LS_0}]_{\beta q}[\lambda_{LS_0}]^*_{\alpha q}$	$[\lambda_{RS_0}]_{\beta f}[\lambda_{RS_0}]^*_{\alpha f}$	$[\lambda_{LS_0}]_{\beta f}[\lambda_{RS_0}]^*_{\alpha f}$	$[\lambda_{RS_0}]_{\beta f}[\lambda_{LS_0}]^*_{\alpha f}$
$S_2^\dagger(lower)$	5/3	2/3(u)	$[\lambda_{LS_2}]_{\beta f}[\lambda_{LS_2}]^*_{\alpha f}$	$[\lambda_{RS_2}]_{\beta f}[\lambda_{RS_2}]^*_{\alpha f}$	$-[\lambda_{LS_2}]_{\beta f}[\lambda_{RS_2}]^*_{\alpha f}$	$-[\lambda_{RS_2}]_{\beta f}[\lambda_{LS_2}]^*_{\alpha f}$
$S_2^\dagger(upper)$	2/3	$-1/3(d)$	0	$[\lambda_{RS_2}]_{\beta f}[\lambda_{RS_2}]^*_{\alpha f}$	0	0
$\tilde{S}_2^\dagger(lower)$	2/3	$-1/3(d_R)$	$[\tilde{\lambda}_{L\tilde{S}_2}]_{\beta f}[\tilde{\lambda}_{L\tilde{S}_2}]^*_{\alpha f}$	0	0	0
$\vec{S}_3^{\,\dagger}(\tau_3 comp)$	1/3	$-2/3$ (u^c)	$[\lambda_{LS_3}]_{\beta f}[\lambda_{LS_3}]^*_{\alpha f}$	0	0	0

Table 2: Parameters for obtaining the $e_\alpha \to e_\beta\gamma$ amplitude, induced by the scalar leptoquarks of the left colomn, using eqns (7) - (10)). α and β are lepton flavour indices, and f is the internal quark index. See figure 3.
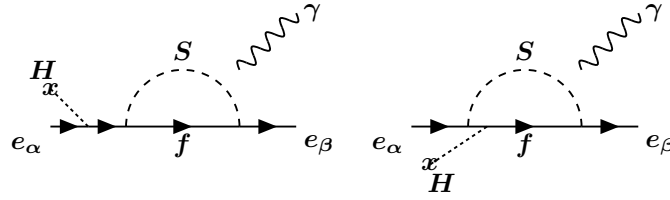

Figure 3: One loop diagrams contributing to $e_\alpha \to e_\beta\gamma$, mediated by a scalar S, and a fermion f. In the diagrams on the left, the scalar S interacts either with doublet or singlet charged leptons, so the Higgs insertion is on an external leg. In the diagrams on the right, S interacts with singlet and doublet charged leptons, so the Higgs insertion can be on the internal lines.

where $x = m_f^2/M_{\tilde{S}}^2 \ll 1$. Inside the square brackets after the first equality, $\rho_{\beta\alpha}$ is from table 2 but not summed over quark flavour q. Assuming x is negligeable, $k \to 1/6$ and $\bar{k} \to 1/12$, which gives the second equivalence; this means that the coefficients of dimension eight operators, in this model, are irrelevant.

In the case where $\widetilde{S_0}$ has generation indices I, one obtains

$$\frac{[\mathbf{S}_R]_{\beta\alpha}}{v^2} \simeq -\frac{1}{4}\sum_I [\tilde{\lambda}_{R\tilde{S}_0}^I \frac{1}{M_{\tilde{S},I}^2}\tilde{\lambda}_{R\tilde{S}_0}^{I\dagger}]_{\beta\alpha} \equiv -\frac{1}{4}[\tilde{\lambda}_{R\tilde{S}_0}D_M^{-2}\tilde{\lambda}_{R\tilde{S}_0}^{\dagger}]_{\beta\alpha} \tag{25}$$

where after the first equality, there is a $[\lambda^I]$ matrix with indices in lepton and quark flavour spaces for each flavour I of leptoquark, and after the second equivalence, the leptoquark index is also implicit ($D_M^2 = \text{diag}\{M_{\tilde{S},1}^2..M_{\tilde{S},3}^2\}$ is the diagonal leptoquark mass matrix). $\lambda_{R\tilde{S}_0}$ is in the charged lepton, down quark and leptoquark mass eigenstate bases, and the quark flavour sum has again been done neglecting quark mass effects $\propto m_q^2/M_{\tilde{S},I}^2$.

Consider now the **singlet S_0 coupling to** $\overline{u^c_{L,R}}e_{L,R}$, without any flavour index of its own, because it is straightforward to add a flavour sum on the leptoquarks to the formulae below. S_0 couples to up-type quarks, so $f = u, c, t$, and the $\mathcal{O}(m_u^2, m_c^2)/M_S^2$ terms in k and $\bar{k}$ can be neglected, as were the quark mass terms in the case of $\widetilde{S_0}$. We keep the top quark mass dependance. For $x = m_f^2/M_S^2$, $t = m_t^2/M_S^2$, one obtains [8]

$$\frac{[\mathbf{S}_R]_{\beta\alpha}}{v^2} = -\sum_f \frac{\rho_{\beta\alpha}}{2M_S^2}\left(-2k(x) + \overline{k}(x)\right)$$

$$\simeq \frac{\left[[\lambda_{RS_0}][\lambda_{RS_0}]^\dagger\right]_{\beta\alpha}}{8M_S^2} - \frac{[\lambda_{RS_0}]_{\beta t}[\lambda_{RS_0}]^*_{\alpha t}}{2M_S^2}\left(\frac{t^3 - 3t^2 + 9t}{4(t-1)^3} - \frac{t(t+2)\ln t}{2(t-1)^4}\right) \tag{26}$$

$$\frac{[\mathbf{S}_L]_{\beta\alpha}}{v^2} = -\sum_f \frac{\omega_{\beta\alpha}}{2M_S^2}\left(-2k(x) + \overline{k}(x)\right)$$

$$\simeq \frac{\left[[\lambda_{LS_0}][\lambda_{LS_0}]^\dagger\right]_{\beta\alpha}}{8M_S^2} - \frac{[\lambda_{LS_0}]_{\beta t}[\lambda_{LS_0}]^*_{\alpha t}}{2M_S^2}\left(\frac{t^3 - 3t^2 + 9t}{4(t-1)^3} - \frac{t(t+2)\ln t}{2(t-1)^4}\right) \tag{27}$$

$$\frac{[\mathbf{T}_R]_{\beta\alpha}}{v^2} = -\sum_f \frac{\xi_{\beta\alpha} m_f}{2M_S^2 v}\left(-2k_f(x) + \overline{k}_f(x)\right)$$

$$\simeq -\frac{[\lambda_{LS_0}]_{\beta f} m_f [\lambda_{RS_0}]^*_{\alpha f}}{2M_S^2 v}\left(\frac{7}{2} + 2\ln\frac{m_f^2}{M_S^2}\right)$$

$$-\frac{[\lambda_{LS_0}]_{\beta t} m_t [\lambda_{RS_0}]^*_{\alpha t}}{2M_S^2 v}\left(\frac{7t^2 + 13t}{2(t-1)^2} - \frac{(2t^3 - 6t^2 + 7t)\ln t}{(t-1)^3}\right) \tag{28}$$

$$\frac{[\mathbf{T}_L]_{\beta\alpha}}{v^2} = -\sum_f \frac{\zeta_{\beta\alpha} m_f}{2M_S^2 v}\left(-2k_f(x) + \overline{k}_f(x)\right)$$

$$\simeq -\frac{[\lambda_{RS_0}]_{\beta f} m_f [\lambda_{LS_0}]^*_{\alpha f}}{2M_S^2 v}\left(\frac{7}{2} + 2\ln\frac{m_f^2}{M_S^2}\right)$$

$$-\frac{[\lambda_{RS_0}]_{\beta t} m_t [\lambda_{LS_0}]^*_{\alpha t}}{2M_S^2 v}\left(\frac{7t^2 + 13t}{2(t-1)^2} - \frac{(2t^3 - 6t^2 + 7t)\ln t}{(t-1)^3}\right) \tag{29}$$

where ρ, ω, ξ and ζ are from table 2 with no sum over q.

3.2.2 Getting invariants

For the $\widetilde{S_0}$, which interacts with charged leptons and down-type quarks, the dipole vertex function is given by eqn (24), or (25), respectively for the cases that $\widetilde{S_0}$ does not, or does, have generation indices. It is an invariant, constructed from spurions from the Lagrangian:

$$\frac{[\mathbf{S}_R]_{\beta\alpha}}{v^2} = -\frac{1}{4}[\tilde{\lambda}_{R\tilde{S}_0} D_M^{-2} \tilde{\lambda}^\dagger_{R\tilde{S}_0}]_{\beta\alpha} \quad , \quad \text{leptoquark } \widetilde{S_0} \text{ with generations} \tag{30}$$

where the quark flavour sum is implicit. Since the down-type quark masses are small compared to $M_{\tilde{S}}^2$, dimension eight and higher operators are irrelevant, the dipole vertex function in this case is unambiguously an invariant.

The dipole vertex functions generated by the leptoquark S_0, which couples to up-type quarks, are not so obviously invariants due to the large top mass. For $\mathbf{S}_{L.R.}$, if $9\lambda_{\beta t}\lambda^*_{\alpha t} m_t^2/M_S^2 \ll [\lambda\lambda^\dagger]_{\beta\alpha}$, the second term in eqns (26) and (27) can be ignored, and the $\mathbf{S}_X$ are approximately the first term, which is an invariant:

$$\frac{\mathbf{S}_X}{v^2} = \frac{\left[[\lambda_{XS_0}][\lambda_{XS_0}]^\dagger\right]_{\beta\alpha}}{8M_S^2} + ... \tag{31}$$

However, if the terms of order m_t^2/M_S^2 should be included, they correspond to dimension eight operators with invariant coefficient $\propto \lambda Y_u Y_u^\dagger \lambda^\dagger$ (in the one-leptoquark-generation case). This ressembles the seesaw case, where the coefficients of dimension eight operators were potentially dangerous, because not neccessarily suppressed by small couplings.

In the presence of λ_L and λ_R, a contribution to $\mathbf{T}$ can arise, propertional to the internal fermion mass, giving the lowest order coefficient proportional to invariants of the form $\lambda_R Y_u^\dagger \lambda_L^\dagger$ or $\lambda_L Y_u \lambda_R^\dagger$. So one obtains

$$\frac{[\mathbf{T}_L]_{\beta\alpha}}{v^2} = -\left(\frac{1}{M_S^2}[\lambda_{RS_0}(c_1 Y_u^\dagger + Y_u^\dagger \ln(Y_u^\dagger Y_u))\lambda^\dagger_{L\tilde{S}_0}]_{\beta\alpha}\right) + ... \quad ,$$

$$\frac{[\mathbf{T}_R]_{\beta\alpha}}{v^2} = -\left(\frac{1}{M_S^2}[\lambda_{LS_0}(c_1 Y_u + Y_u \ln(Y_u Y_u^\dagger))\lambda^\dagger_{R\tilde{S}_0}]_{\beta\alpha}\right) + ... \quad , \quad S_0 \text{ with } \lambda_L, \lambda_R \tag{32}$$

where $c_1 = 7/4 + \ln(v^2/M_S^2)$, and the quark flavour sum is implicit. The logarithm is unfortunate, from the perspective of reconstructing the new flavour structures, because one needs its coefficient to reconstruct the new flavoured matrices. For instance, imagine that $[\lambda_L]_{\beta q}$ was determined in some other process, and that the pattern of $e_\alpha \to e_\beta \gamma$ decays indicates that they are mediated by a hierachical flavour tensor with chirality flip on the internal line. Then to obtain λ_R from $e_\alpha \to e_\beta \gamma$ data requires knowing the coefficient of the logarithm. Two positive feature of logarithms are that one can often guess their presence, and, as discussed in section 3.2.3.1 of [10], if one knows the coefficient, the logarithm does not obstrct the reconstruction of new flavour structures.

Eqn (32), like eqn (31), neglects the dimension eight terms of order $\lambda_R Y_u Y_u^\dagger Y_u \lambda_L v^2/M_S^4$ and $\lambda_L Y_u Y_u^\dagger Y_u \lambda_R v^2/M_S^4$, which are not small for the top Yukawa.

Finally, it is clear that the dipole coefficient induced by S_0 will only be proportional to a single invariant, if one of $\mathbf{S}_{L.R}$, $\mathbf{T}_L$ or $\mathbf{T}_R$ dominates over the others. See the comparaison of their relative sizes in section 4.3.2.

4 Testing hierarchies?

It is well-known, that if the flavour tensors $\mathbf{S}_{L.R}$ or $\mathbf{T}$ are dominated by their largest eigenvalue, then they are compartively predictive. The idea is that, if one of the $\tau \to e_\beta \gamma$ decays is observed, then the current bound on $\mu \to e\gamma$ implies a small mixing angle in the diagonalisation of the flavour tensor, and this small angle suppresses the other $\tau \to \ell'\gamma$ decay. This section is somewhat independent of the previous section, because the flavour tensors contributing to the vertex function can be hierarchical without being invariants. Nonetheless, subsection 4.3 discusses when hierarchical invariants are obtained, because ideally, one wants to reconstruct the spurions in the Lagrangian from measurements of the vertex function.

4.1 Defining hierarchies

We define a matrix, in particular the flavour tensors $\mathbf{S}_{L.R}$, $\mathbf{T}_L$ and $\mathbf{T}_R$, to be "hierarchical", when the off-diagonal elements are dominated by the largest eigenvalue. A hermitian matrix $\mathbf{S}$ (for instance $\mathbf{S}_R$ or $\mathbf{S}_L$),can be written

$$\mathbf{S} = V^\dagger D_S V \quad , \quad D_S = \text{diag}\{s_1, s_2, s_3\} \quad , \quad s_1 \le s_2 \le s_3 \tag{33}$$

and s_3 will give the largest contribution to $S_{\beta\alpha}$, when

$$V_{1\beta}^* s_1 V_{1\alpha} \ , \ V_{2\beta}^* s_2 V_{2\alpha} \ll V_{3\beta}^* s_3 V_{3\alpha} \tag{34}$$

Eqn (34) is satisfied for eigenvalues s_1, s_2 and s_3, if the mixing angles are large enough [**]:

$$V_{j\beta} \gtrsim \sqrt{\frac{s_i}{s_j}} V_{i\beta} \quad i \le j \quad . \tag{35}$$

This relates elements of a colomn in V: the bottom row $[V]_{3\alpha}$ has to be bigger than a fraction $\sqrt{s_i}$ of the elements in the colomn above it.

The flavour tensor $\mathbf{T}_R = \mathbf{T}_L^\dagger$, like a Yukawa matrix, can be diagonalised with independent unitary matrices on the left and right:

$$\mathbf{T}_R = V_L^\dagger D_T V_R \quad , \quad D_T = \text{diag}\{t_1, t_2, t_3\} \quad , \quad t_1 \le t_2 \le t_3 \tag{36}$$

If the matrix elements of V_L and V_R satisfy eqn (35) (replacing $s_i \to t_i$), then, just as in the case of $\mathbf{S}_{L.R}$, the off-diagonal elements $[\mathbf{T}_L]_{\beta\alpha}$, and $[\mathbf{T}_R]_{\beta\alpha}$ will be dominated by t_3.

4.2 Observing a hierarchy

Suppose initially that New Physics induces only one of $\mathbf{S}_L$ or $\mathbf{S}_R$ (the possible presence of $\mathbf{S}_L$ and $\mathbf{S}_R$, or $\mathbf{T}_{L,R}$ will be discussed later). Then we start with two assumptions
1) the flavour tensor $\mathbf{S}_X$ is hierarchical
2) one of the $\tau \to e_\beta \gamma$ decays is observed at Super-B factories. This is neccessary for them to be able to test anything. To be concrete, suppose that $\tau \to \mu\gamma$ is observed.
Combined with table 1, these assumptions fix

$$3 \times 10^{-3} \lesssim |V_{3\tau}^* s_3 V_{3\mu}| \lesssim 10^{-3} \tag{37}$$

where $s_3 \sim \lambda^2 v^2 / M_{BSM}^2 \le v^2 / M_{BSM}^2$ (assuming that $\mathbf{S}_{L.R}$ is induced at dimension six by New Particles with couplings $\lambda \le 1$). In addition, the non-observation of $\mu \to e\gamma$ puts an upper bound

$$\frac{\widetilde{BR}(\mu \to e\gamma)}{\widetilde{BR}(\tau \to \mu\gamma)} = \frac{|S_{e\mu}|^2}{|S_{\mu\tau}|^2} = \frac{|V_{3e}|^2}{|V_{3\tau}|^2} \lesssim 10^{-4} \quad . \tag{38}$$

With this bound on V_{3e}, the only way that $\tau \to e\gamma$ could detectable rate at Super-B factories, is if $V_{3\mu} \simeq V_{3e}$ are both small. However, from eqn (37), this would require $s_3 \ge .1$, or $M_{BSM} \lesssim 500$ GeV, which is a low scale for new particles with $\mathcal{O}(1)$ couplings to charged leptons[††]. If we make the further assumption

3) that $s_3 \sim \dfrac{\lambda^2 v^2}{M_{BSM}^2} \ll .1$, that is, $M_{BSM} \gtrsim 500$ GeV

[**]Equation (35) implies (34), but since eqn (34) is a lower bound on a product of elements of V, it can be satisfied when one of the V_{ij} does not satisfy eqn (35).

[††]The 7 TeV LHC, with the current 5 fb^{-1} of data, is surely sensitive to leptoquarks of mass $\lesssim 600$ GeV —even if they decay to τt [41] — and possibly also to heavy singlet fermions

then we can conclude that Super-B factories can falsify the assumption of hierarchical couplings in $\mathbf{S}_L$ or $\mathbf{S}_R$, by detecting both $\tau \to \mu\gamma$ and $\tau \to e\gamma$.

An alternative way to see the $\mu \to e\gamma$ bound, for a hierarchical flavour tensor, is illustrated in figure 4. The paremeter space of the three elements $|V_{3\alpha}|^2$, who satisfy $\sum_\alpha |V_{3\alpha}|^2 = 1$, is the red equilateral triangle. The blue hyperbola corresponds to $\Gamma(\tau \to \mu\gamma) \propto |V_{3\mu}^* V_{3\tau}|^2$ visible at a Super-B Factory. Then the green plane corresponds to the bound of eqn (38); allowed points live on the intersection of the red and blue surfaces, and on the far side of the green plane. The hyperbola corresponding to a fixed rate for $\tau \to e\gamma$ is not drawn (it would be in the vertical and $|V_{3e}|^2$ plane, intersecting the blue hyperbola). The green and red planes are independent of the (unknown) eigenvalue s_3, which is $\propto v^2/M_{BSM}^2$. The hyperbolae do depend on s_3, and move away from the axes as s_3 decreases (or as the branching ratio increases for fixed s_3). For the value of s_3 chosen in this plot, it is clear that the $\tau \to e\gamma$ hyperbola corresponding to a rate similar to $\tau \to \mu\gamma$, would live on the near side of the green plane, so a Super-B Factory cannot see $\tau \to e\gamma$ and $\tau \to \mu\gamma$.
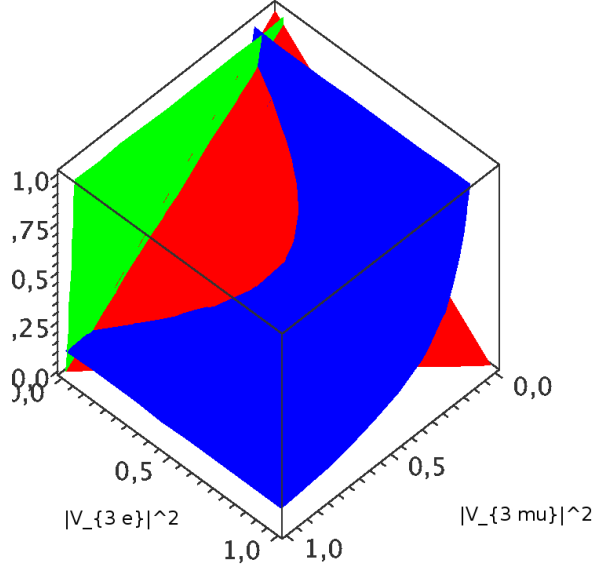

Figure 4: A graphical representation of the claim that a hierarchical flavour tensor (see section 4.1) predicts that a Super-B factory should not see $\tau \to e\gamma$ and $\tau \to \mu\gamma$. The axes are $\{|V_{3e}|^2, |V_{3\mu}|^2, |V_{3\tau}|^2\}$, which satisfy $|V_{3e}|^2 + |V_{3\mu}|^2 + |V_{3\tau}|^2 = 1$ represented by the red triangle. The vertical axis is $|V_{3\tau}|^2$. The blue hyperbola corresponds to $\Gamma(\tau \to \mu\gamma) \propto |V_{3\mu}^* V_{3\tau}|^2$ visible at a Super-B Factory. The green plane corresponds to the $\mu \to e\gamma$ bound of eqn (38). Allowed points live on the intersection of the red and blue surfaces, and on the far side of the green plane. A hyperbola (not drawn) corresponding to $\tau \to e\gamma$ at a rate similar to $\tau \to \mu\gamma$ would live on the near side of the green plane, see discussion after assumption 3), so a Super-B Factory should not see $\tau \to e\gamma$ and $\tau \to \mu\gamma$.

The assumption that only one of $\mathbf{S}_L$ or $\mathbf{S}_R$ is present is minimal, and perhaps justified by the absence to date of new TeV-scale particles. However, it can in principle be tested: if the decaying lepton is polarised, then by angular momentum conservation, the angular distribution of the final lepton will depend on its chirality [9]. Therefore, if $\tau \to \mu\gamma$ and $\tau \to e\gamma$ are both observed at Super-B factories (where polarised τs may be possible[6, 7]), one can hope to determine, from the final state angular distributions, whether they are induced by $\mathbf{S}_L$ and/or $\mathbf{S}_R$. If the final state angular distributions are different, then this is compatible with hierarchical $\mathbf{S}_L$ and $\mathbf{S}_R$; if both $\tau \to \mu\gamma$ and $\tau \to e\gamma$ have the same final state angular distributions, then this is incompatible with hierarchical couplings in $\mathbf{S}_{L.R}$. So one more condition is required to test a hierarchy in the $\mathbf{S}_{L.R}$:

4) in decays of polarised τs, the angular distribution of final state leptons can be measured

However, New Physics which induces $\mathbf{S}_L$ and $\mathbf{S}_R$ may also induce $\mathbf{T}_X$, as was the case for the leptoquark S_0. Consider a hierarchical $\mathbf{T}$, neglecting at first the $\mathbf{S}_{L.R}$. The $e_\alpha \to e_\beta\gamma$ branching ratios will therefore be parametrised by t_3, and the third rows of V_L and V_R. This makes five parameters (as opposed to three for $\mathbf{S}_L$ or $\mathbf{S}_R$), so the polarised branching ratios are required to hope to test the hierarchical pattern in $\mathbf{T}$. Suppose, for instance, that $\tau_L \to \mu_R\gamma$ is

$\widetilde{BR}$	**S**$_L$	**S**$_L$	**S**$_R$	**S**$_R$	**T**	**T**	**T**	**T**
$\tau_R \to \mu_L\gamma$	$\checkmark$	x	x	x	$\checkmark$		x	
$\tau_L \to \mu_R\gamma$	x	x	$\checkmark$	x		$\checkmark$		x
$\tau_R \to e_L\gamma$	x	$\checkmark$	x	x		x		$\checkmark$
$\tau_L \to e_R\gamma$	x	x	x	$\checkmark$	x		$\checkmark$	

Table 3: If the polarised decay indicated by $\checkmark$ is observed, then a hierarchical pattern in the operators listed in the top row predicts that the decays indicated by x should not be seen at Super-B factories. The blank spaces mean that no prediction is made. This table depends on the assumptions 1-4 listed in the text, see in particular assumption 3). The operators $\mathbf{S}_L$ and $\mathbf{S}_R$ can induce either of their two colomns, $\mathbf{T}$ can induce any two non-conflicting colomns. See the discussion at the end of section 4.2.

measured. Both $\widetilde{BR}(\mu_L \to e_R\gamma)$ and $\widetilde{BR}(\mu_R \to e_L\gamma)$ are small, so, in particular

$$\frac{\widetilde{BR}(\mu_R \to e_L\gamma)}{\widetilde{BR}(\tau_L \to \mu_R\gamma)} = \frac{|T_{e\mu}|^2}{|T_{\tau\mu}|^2} = \frac{|V_{L3e}^* V_{R3\mu}|^2}{|V_{L3\tau}^* V_{R3\mu}|^2} \lesssim 10^{-4} \tag{39}$$

so with assumption 3) above, $\widetilde{BR}(\tau_R \to e_L\gamma) \propto |V_{L3e}^* V_{R3\tau}|^2$ should be beyond the reach of Super-B factories. Similarly, if $\tau_R \to e_L\gamma$ was measured, a hierarchical structure in $\mathbf{T}$ predicts that $\tau_L \to \mu_R\gamma$ is inaccessible.

Table 3 summarises which processes should not be seen, given a polarised decay induced by a particular flavour tensor. For instance, if one $\tau \to e_{X\beta}\gamma$ decay is observed, this is consistent with a hierarchical $\mathbf{S}_X$ or $\mathbf{T}$. If both one of the $\tau_R \to e_{L\beta}\gamma$, and one of the $\tau_L \to e_{R\beta}\gamma$ are observed, this is consistent with simultaneous hierarchical $\mathbf{S}_R$ and $\mathbf{S}_L$. If the particular combinations, $\tau_R \to \mu_L\gamma$ and $\tau_R \to e_L\gamma$, or $\tau_L \to \mu_R\gamma$ and $\tau_L \to e_R\gamma$, are observed, this is consistent with a hierarchy in $\mathbf{T}$. So hierarchies in the various flavour tensors predict that various rates are suppressed. However, if all the $\tau \to e_\beta\gamma$ decays are observed in all polarisations, this is consistent with the simultaneous presence of hierarchical $\mathbf{S}_{L.R}$ and $\mathbf{T}$. That is, a hierarchical pattern in $\mathbf{T}$ or in the $\mathbf{S}_{L.R}$ make opposite predictions: if, for instance, $\tau_R \to \mu_L\gamma$ is observed, this is compatible with $\mathbf{S}_L$, in which a hierarchy imposes that $\tau_R \to e_L\gamma$ must not be seen. However, $\tau_R \to \mu_L\gamma$ could also be induced by $\mathbf{T}$, where a hierarchy is consistent with $\tau_R \to e_L\gamma$, but not $\tau_L \to e_R\gamma$. Therefore, with the assumptions made so far, the hierarchical structure cannot be falsified. In addition, one must assume that

5) $either$ the $\mathbf{S}_{L.R}$, or $\mathbf{T}$ is dominant.

Then it is clear from the table that a hierarchical structure can be falsified: for instance, if $\tau_L \to \mu_L\gamma$ and $\tau_L \to e_L\gamma$ are observed, this is inconsistent with a hierarchy in $\mathbf{S}_L$ (or $\mathbf{S}_R$).

4.3 Hierarchies and invariants

This subsection makes contact between flavour tensors as invariants, and the prospects of testing their hierarchical structure. Section 4.3.1 discusses the conditions which the spurions should satisfy, such as to obtain a hierarchical invariant. Section 4.3.2 uses invariants to study whether the leptoquark S_0, which can induce $\mathbf{S}_{L.R}$ and $\mathbf{T}$, satisfies assumption five for testing a hierarchy.

4.3.1 Getting a hierarchy in $\mathbf{S}_{L.R}$

Ideally, the hierarchy condition eqn (35) could be translated to conditions which the spurions should satisfy to ensure a hierarchical invariant. However, this phenomenological approach amounts to expressing elegant invariants in explicit basis-dependent form, which defeats the purpose of invariants, and risks to produce opaque interlinked bounds on unknown eigenvalues and mixing angles. Instead, it is simple to study the issue in the wrong direction, that is, start from hierarchies in the spurions, and enquire when then are transmitted to the flavour tensors.

Two simple cases are:

1. degenerate eigenvalues M_I of the matrix M of the New Particles
   Consider to be concrete the seesaw case. If λ is written, in the charged lepton and heavy singlet mass eigenstate

bases, as $V_L^\dagger D_\lambda V_R$, then for degenerate singlet mass -squared $|M_I|^2$, the hierarchy condition on S eqn (35) is trivially applied to $\lambda^\dagger \lambda$ in the charged lepton mass eigenstate basis:

$$[V_L]_{j\beta} \geq \frac{\lambda_i}{\lambda_j}[V_L]_{i\beta} \quad i \leq j \tag{40}$$

To explore the area around the degenerate mass limit, one can write

$$[\lambda D_M^{-2}\lambda^\dagger]_{\beta\alpha} = \frac{1}{M_1^2}\left([\lambda\lambda^\dagger]_{\beta\alpha} - [\lambda]_{\beta J}\frac{M_1^2 - M_J^2}{M_J^2}\lambda_{\alpha J}^*\right) \tag{41}$$

and assume that the first term satisfies eqn (35). Then the second term is negligeable if

$$M_J^2 - M_1^2 \ll M_J^2 \tag{42}$$

2. flavour-independent vertices
   If, in general, $\lambda \equiv V_L^\dagger D_\lambda V_R$, then for D_λ proportional to the identity matrix, the hierarchy condition of eqn (35) applies to $V = V_L^\dagger V_R$ and D_M^{-2}.

Illuminating formulae for the case where there are hierarchies in λ and M are not obvious to find.

For $\widetilde{S_0}$, without generations indices, $\mathbf{S}_R$ will have hierarchical form if $\widetilde{\lambda}_R^* \widetilde{\lambda}_R^T \equiv V_e D_{\widetilde{\lambda}}^2 V_e^\dagger$ is hierarchical on its lepton indices (equivalently, if V_e and $D_{\widetilde{\lambda}}^2$ satisfy conditions (35)). If $\widetilde{S_0}$ has generation indices, then for three leptoquarks of mass squared M_I^2, the same conditions as discussed for the seesaw are sufficient to obtain a hierarchical flavour tensor. Similar comments would apply to the leptoquark S_0 with couplings λ_L or λ_R.

4.3.2 T versus $\mathbf{S}_{L.R}$ for the S_0 leptoquark

The fifth condition for testing/confirming a hierarchy, is that only one of $\mathbf{T}$ or $\mathbf{S}_{L.R}$ makes a significant contribution to $\tau \to e_\beta \gamma$. The leptoquark S_0, with both couplings λ_L and λ_R, induces both structures of matrix element, so it is interesting to compare the relative size of $\mathbf{T}$ and $\mathbf{S}_{L.R}$.

First, notice that constraints from two-quark-two-lepton contact interactions apply directly to the magnitude of terms contributing to the invariant. This is a useful feature of invariants. The bounds from $\tau \to e_\beta \pi$ [38] impose that the u quark contribution to $[\mathbf{S}_{L.R}]_{\beta\tau}$ is less than 2×10^{-4}, so insufficient to generate an observable $\tau \to e_\beta \gamma$ rate (see eqn (37)). On the other hand, a diagram with c quarks in the loop can induce an observable $\tau \to e_\beta \gamma$ via $\mathbf{S}_{L.R}$ or $\mathbf{T}$; there are $\mathcal{O}(1)$ bounds on $[\mathbf{S}_R]_{\beta\tau}$ fron $Z \to \overline{e_\beta}\tau$, contraints from measuring V_{cs} on $[\mathbf{S}_L]_{\beta\tau}$ which are an order of magnitude smaller, and finally bounds from $D_s \to \tau\nu_\beta$ on the c-loop contribution to $\mathbf{T}$ which (just) allow it to contribute an observable $\tau \to e_\beta \gamma$ rate. Finally, it is clear that loops involving a top quark can contribute to an observable rate via at most two of $\mathbf{S}_{L.R}$ and $\mathbf{T}$. This is because one cannot simultaneously obtain $[\lambda_L]_{\tau t}^*[\lambda_L]_{\beta t}m_\tau \sim [\lambda_R]_{\tau t}^*[\lambda_R]_{\beta t}m_\tau \sim [\lambda_L]_{\tau t}^*[\lambda_R]_{\beta t}m_t \sim [\lambda_R]_{\tau t}^*[\lambda_L]_{\beta t}m_t$. So unfortunately, S_0 leptoquarks interacting principally with a c quark could have hierarchical couplings that generate similar rates for the four processes $\tau \to e_{L,R}\gamma$ and $\tau \to \mu_{L,R}\gamma$. This is illustrated in figure 5, which shows flat asymmetries $A_{e\mu}$(solid) and Ae_{LR} (dotted), defined as

$$A_{e\mu} \equiv \frac{\widetilde{BR}(\tau \to \mu\gamma) - \widetilde{BR}(\tau \to e\gamma)}{\widetilde{BR}(\tau \to \mu\gamma) + \widetilde{BR}(\tau \to e\gamma)} \quad , \quad Ae_{LR} \equiv \frac{\widetilde{BR}(\tau \to e_L\gamma) - \widetilde{BR}(\tau \to e_R\gamma)}{\widetilde{BR}(\tau \to e\gamma)} \quad . \tag{43}$$

An asymmetry for muons $A\mu_{LR}$ can also be defined; it has the same distribution as Ae_{LR}. The histograms are obtained by varying the six couplings $\lambda_{L\ell c}, \lambda_{R\ell c}$ (for $\ell \in \{\tau, \mu, e\}$) between .01 and 1 with a log prior, and computing the asymmetries for all the points which give a total rate for $\tau \to e_\beta \gamma$ below the current bounds and within the reach of SuperB Factories. The normalisation of the vertical axis is arbitrary.

5 Discussion

The observed neutrino masses demonstrate the presence of New Physics in the lepton sector. However, we do not know what it is. One approach to this problem is to calculate in motivated models, thereby restricting their parameters and identifyng their predictions. Alternatively, one can try to "reconstruct" the New Physics from data. This paper aims to follow the second, more phenomenological philosophy.
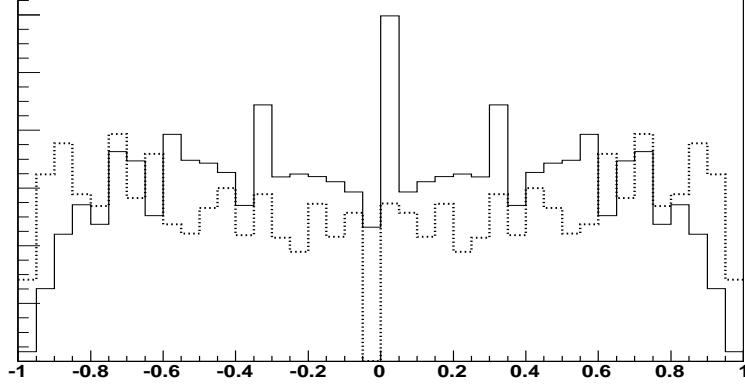
Figure 5: This plot illustrates the importance of assumption 5) for testing hierarchies. An S_0 leptoquark coupled principally to c quarks would induce hierarchical flavour tensors, but can give similar rates to the four final states $e_L\gamma$, $e_R\gamma$ $\mu_L\gamma$, and $\mu_R\gamma$. The solid and dotted lines are respectively the asymmetries $A_{e\mu}$ and Ae_{LR} (see eqn (43)), obtained after scanning over allowed couplings (see text around eqn (43)).

5.1 Invariants

If the masses and couplings constants of the new particles were known, the dipole "vertex functions" $\mathbf{X}_{L,R}$ of eqn (1), which are observable, could be calculated to arbitrary accuracy. However, since we know neither the new particle identities, nor their mass scale, nor the structure of their flavoured couplings, it would be convenient to be able to separate these unknowns in $\mathbf{X}_{L,R}$. So the first aim of this paper was to study, for a variety of new particle contents, whether it was possible to factorize $\mathbf{X}_{L,R}$ as the product of a dimensionful, unflavoured constant, multiplying an "invariant". We refer to flavoured coupling matrices from the Lagrangian as "spurions", and define invariants to be products of spurions. Since $\mathbf{X}_{L,R}$ is a dimensionful matrix in flavour spaces, it can always be written as a dimensionful factor multiplying a dimensionless tensor in flavour spaces, which we call a "flavour tensor" (see eqn (3). The crucial question is whether this flavour tensor is an invariant. This is an interesting question for phenomenology, because it is comparatively simple to obtain information about spurions (the Lagrangian flavour structures) from invariants.

In this paper, we only consider one loop coontributions in New Physics models without a conserved parity. This means that in the loop, there is always a Standard Model particle, and a new particle. In such models, it seems that the question about invariants reduces to whether the $\mathbf{X}_{L,R}$ are dominated by the coefficient of the dimension six operator of eqn (14). This claim is based on comparing the formulae for the $\mathbf{X}_{L,R}$, to the Effective Field Theory expansion in $1/\Lambda_{NPFC}$, where Λ_{NPFC} is the scale of new flavour changing physics in the lepton sector. In eqn (3), the $\mathbf{X}_{L,R}$, of mass dimension -1, are written as a sum of dimensionful constants $\sim 4G_F v/(16\pi^2\sqrt{2})$, multiplying dimensionless flavour tensors. The flavour tensors relevant to $e_\alpha \to e_\beta\gamma$, are $\mathbf{S}_{L,R}$ and $\mathbf{T}_{L,R}$, which respectively enccde flavour change among leptons of the same SU(2) representation, or chirality-changing flavour change. Formulae for these flavour tensors are in eqns (7) to (13). They involve dimensionless k and y functions of the mass ratio in the loop, which are given in the Appendix. Assuming that the new particle masses are at the scale $\Lambda_{NPFC} > v,$, these k and y functions can be expanded in v^2/Λ^2_{NPFC}, because one of the masses is a Standard Modele mass. The coefficients in this expansion will be combinations of coupling constants. The terms in this expansion can be identified to an Effective Field Theory [42, 43] expansion in $1/\Lambda_{NPFC}$, of operators which can contribute to the dipole interaction. These include the usual dipole operator (eqn (14) and first term below) at $\mathcal{O}(1/\Lambda^2_{NPFC})$, plus gauge invariant operators with an arbitrary number of extra Higgs fields:

$$\frac{\mathbf{C}^{(6)}_{\beta\alpha}}{\Lambda^2_{NPFC}}(\overline{\ell}_\beta H)\sigma^{\mu\nu}P_R e_\alpha F_{\mu\nu} + \frac{\mathbf{C}^{(8)}_{\beta\alpha}}{\Lambda^4_{NPFC}}H^\dagger H(\overline{\ell}_\beta H)\sigma^{\mu\nu}P_R e_\alpha F_{\mu\nu} + ... \tag{44}$$

In this way of thinking, $\mathbf{C}^{(6)}$ is our invariant, and it will be a good approximation to the dipole vertex functions if higher order terms in eqn (44) can be neglected, as would be natural for $\Lambda_{NP} \gg v$. However, the New Physics scale should be $\lesssim 10$ TeV [‡‡] to see $\tau \to e_\beta\gamma$ at a Super-B Factory.

[‡‡]Normally, for such "TeV-scale" New Physics, the coefficient $\mathbf{C}^{(6)}$ is matched to the dipole vertex function at the scale where the new particles are removed from the theory.

Two models were considered, to explore the possibility that the coefficient matrix $\mathbf{C}^{(6)}$ is an invariant, and an adequate approximation to the dipole vertex function. In the case of a non-SUSY, TeV-scale type I seesaw, for which the Lagrangian is given in eqn (16), the new physics interacts with the doublet leptons, so contributes to $\mathbf{S}_L$. Two types of singlet scalar leptoquark were also considered, respectively coupled to down or up-type quarks. If a generation number is attributed to the leptoquarks, then the coupling constants at the vertices on either side of the loop are three index tensors, and the invariant is obtained by summing generation on both internal lines. At $\mathcal{O}(1/\Lambda^2_{NPFC})$, the flavour tensors $\mathbf{S}_{L.R}$ due to leptoquarks or the seesaw are proportional to the invariant

$$\lambda_2 \mathbf{M}^{-2} \lambda_1^\dagger$$

(see eqns (21),(30), and (31)), where $\mathbf{M}^2$ is the mass-squared matrix of the new particles, and λ_1 and λ_2 are the new couplings appearing in the left and right side of the blobs on the top row of figure 1 (In the models considered in this paper, $\lambda_1 = \lambda_2$).

The leptoquark S_0 could also interact with singlet and doublet fermions via the coupling matrices λ_R and λ_L. The chirality flip neccessary for the dipole can arise on the internal quark line, allowing S_0 to generate the flavour tensor $\mathbf{T}$ represented in the second row of figure 1. This gives invariants of the form

$$\lambda_L \mathbf{Y}_u \mathbf{M}^{-2} \lambda_R^\dagger \quad , \quad \lambda_R \mathbf{Y}_u^\dagger M^{-2} \lambda_L^\dagger$$

(see eqn (32)). From these cases, we can extrapolate that the flavour structure of $\mathbf{C}^{(6)}$ is obtained by multiplying one SM Yukawa matrix with two new flavoured couplings matrices, and possibly an inverse mass-squared matrix for flavoured new particles (a dimensionless invariant can still be obtained by normalising by the smallest flavoured new particle mass^2 M_1^2). The form of the invariant can be obtained by inspecting the relevant Feynman diagrams: vertices give a coupling constant, and heavy propagators give $\mathbf{M}^{-2}$.

It remains the question of whether the vertex functions $\mathbf{X}_{L,R}$ can reasonably be replaced by the coefficient $\mathbf{C}^{(6)}$ of dimension six operators, which is proportional to an invariant. As discussed after eqn (44), there are corrections proportional to v^2/Λ^2_{NPFC}, which is < 1 by assumption. The perturbative expansion is therefore well defined. However, in flavour physics, there can be small mixing angles and hierarchical couplings, so the relevant question is whether the $\mathcal{O}(v^2/\Lambda^2_{NPFC})$ terms we wish to drop are small compared to the invariant that was retained. The answer is model-dependent. In the case of the leptoquark $\widetilde{S}_0$, the $\mathcal{O}(v^2/\Lambda^2_{NPFC})$ terms are multiplied by a down-type Yukawa coupling squared, and it is reasonable to neglect corrections $\lesssim m_b^2/M_{\widetilde{S}_0}^2$. However, in the seesaw model, the $\mathcal{O}(v^2/\Lambda^2_{NPFC})$ corrections are $\sim m_W^2/M^2$ (for M a TeV-scale singlet neutrino mass), and for the leptoquark S_0, they are $\sim m_t^2/M_{S_0}^2$. Such corrections could contribute an observable $\tau \to e_\beta \gamma$ rate for new particle masses $\lesssim 2$ TeV (see eqns (4) and (5)).

There can also arise logarithms of flavoured mass matrices, as for example in eqn (32). These could induce errors in the reconstruction of a flavoured matrix Y, because the eigenvalues of Y and $Y(1 + \log Y)$ are different, even though they are diagonalised by the same matrix. This is unfortunate. However, maybe logarithmic cirrections can be considered "small" in a first approximation.

Another stumbling block, in approximating the dipole vertex function as an invariant multiplied by an unknown constant, is that several distinct invariants could contribute to $\mathbf{C}^{(6)}$. This was the case for the S_0 leptoquark, which could induce $\mathbf{S}_{L,R}$, $\mathbf{T}_L$ and $\mathbf{T}_R$ (see eqns (26) to (29)). If new spurions are few, or if new particles are rare, this problem may not arise. However, as studied in section 4.3, the leptoquark S_0 can induce all four flavour tensors with similar contributions to $\tau \to e_\beta \gamma$, while remaining consistent with phenomenological constraints. It could be interesting to study whether there is a unique invariant in models, such as the supersymmetric seesaw, with a single new spurion for the charged leptons (the neutrino Yukawa matrix), but several new particles.

Finally, the invariant could be interpreted as a term from an MFV expansion, as these terms are also constructed by multiplying flavoured matrices from the Lagrangian ("spurions"). However, there are many terms in an MFV expansion, multiplied by arbitrary $\lesssim 1$ constants. The aim here is to obtain only one invariant, in the hope of reconstructing the spurions, so some justification for rejecting more complicated invariants is required. We suppose that sums over flavour indices arise from loops, so are accompagnied by a $1/(16\pi^2)$, which could suppress invariants containing more "spurions". In addition, Yukawa matrices have a Higgs leg, which may be contracted in a loop, which naively gives an additional $1/(16\pi^2)$ suppression. Alternatively, if the Higgs leg remains as a vev, this brings a factor of v/Λ_{NP}. In summary, considering only the simplest invariants may be justified, provided that its reasonable to neglect higher order terms in both the loop expansion, and the Effective Field Theory expansion in v/Λ_{NP}.

5.2 Testing a hierarchy?

The current bound from $\mu \to e\gamma$ of

$$\frac{\Gamma(\mu \to e\gamma)}{\Gamma(\mu \to e\nu\bar{\nu})} \leq 2.4 \times 10^{-12}$$

is almost four orders of magnitude more restrictive than the planned sensitivity of SuperB Factories to $\tau \to e\gamma$ and $\tau \to \mu\gamma$ (see table 1). Pessimists could interpret that the coefficients of lepton flavour violating operators are suppressed, so $\tau \to e\gamma$ and $\tau \to \mu\gamma$ will be beyond the reach of Super B Factories. This paper takes the opposite view that SuperB Factories will see $\tau \to e\gamma$ and/or $\tau \to \mu\gamma$. From a model-building perspective, this could arise because flavour change is larger in the third generation. From a phenomenological perspective, one can only learn about New Physics by seeing it, so to address the question "what can the $e_\alpha \to e_\beta\gamma$ decays tell us about flavour structure in the lepton sector?", we assume that $\tau \to e\gamma$ and/or $\tau \to \mu\gamma$ are observed. Furthermore, the smallest angle in the CKM matrix is $V_{ub} \sim 4 \times 10^{-3}$, so a 10^{-4} suppression due to a small mixing angle squared is "standard".

In practise, we study the less ambitious question of whether a hierarchical flavour tensor makes falsifiable predictions, where the definition of hierarchical is given in section 4.1. It is model-independent, and unrelated to the invariants discussed in the first part of the paper. We focus on a hierarchical structure, because it has sufficiently few parameters to be predictive, and because the Yukawa matrices are hierachical. A hierarchical flavour tensor which induces $\tau \to e\gamma$ or $\tau \to \mu\gamma$ at a Super-B Factory, must contain a small parameter to suppress $\mu \to e\gamma$ below the current bound. Section 4 presents the argument that this small parameter also suppresses a $\tau \to e_\beta\gamma$ decay, implying that a hierarchical flavour tensor forbids the observation of all the modes $\tau \to e_L\gamma$, $\tau \to e_R\gamma$, $\tau \to \mu_L\gamma$ and $\tau \to \mu_R\gamma$. This argument rests upon five assumptions, given in section 4.2. The third and fifth appear problematic. The third assumption is that new particle mass scale is $\gtrsim 600$ GeV, so that the couplings which control the observed $\tau \to e_\beta\gamma$ rate are "large". This enforces a hierarchy with respect to the couplings which control $\mu \to e\gamma$, and the other $\tau \to e'_\beta\gamma$ decay. The LHC may verify this assumption in several models. The fifth assumption is that the dipole vertex function should not have its general form given in eqn (3), but rather should be dominated by either $\mathbf{S}_{L.R}$ or $\mathbf{T}$. This means that New Physics should be in one of the doublet or singlet lepton sectors (generating $\mathbf{S}_{L.R}$), or, if it interacts with both, then the amplitudes with chirality flip inside the loop ($\mathbf{T}$) should dominate those with chirality flip on the decaying fermion line ($\mathbf{S}_{L.R}$). This condition arises because, for a given observed $\tau \to e_\beta\gamma$ process, $\mathbf{S}_{L.R}$ and $\mathbf{T}$ predict different rates to be suppressed, as is summarised in table 3. An example where this condition is not satisfied is given in section 4.3.2. However, if new particles below 10 TeV are rare, this fifth assumption is not unreasonable.

Finally, if a hierarchy were measured, then it could be interesting to suppose that the flavour tensor is an invariant, and explore what this implies about the flavoured matrices of the Lagrangian. Section 4.3.1 briefly discusses the simpler question of when a hierarchy in the Lagrangian flavour structures is transmitted to the flavour tensors contributing to the dipole vertex function.

5.3 Summary

Beyond the Standard Model physics is required in the lepton sector for neutrino masses. If present below 10 TeV, it could induce observable $e_\alpha \to e_\beta\gamma$ decays in upcoming experiments. Observing such decays could give information on the flavour structure of this New Physics.

A hierarchical (in flavour space) dipole vertex function (where hierarchical means dominated by its largest eigenvalue), could be confirmed if some, but not all of the $\tau \to e_\beta\gamma$ decays are observed. Were it possible to distinguish internal from external line chirality flip, such a hierarchy could be tested.

In simple models (with few loop diagrams containing one SM particle), the coefficient of the $\mathcal{O}(1/M_{BSM}^2)$ term in the dipole vertex function is a "Jarlskog-like invariant", meaning that it is obtained by multiplying flavoured matrices from the Lagrangian. It has the form $\lambda_1 M_{BSM}^{-2} \lambda_2^\dagger$, where λ_1 and λ_2 are the flavoured couplings on the two sides of the loop. The invariant is a better approximation to the vertex function when the particles in the loop are (very) weakly coupled to the Higgs. Invariants elegantly give a linear relation to flavoured matrices in the Lagrangian, avoid confusion about basis, and may simplify the combination of bounds from various processes on New Physics.

Acknowledgements

Appendix: Summary of results from Lavoura [8]

Lavoura defines coefficients σ_L, σ_R, related as

$$\frac{e}{16\pi^2} X_L = \frac{i\sigma_L}{2}$$

to the $\mathbf{X}_X$ used here. The σ_X are written in terms of functions k_i, k_f, etc, which are $m_i \times$ the definitions given here (m_α is already scaled out of the definition of $\mathbf{S}_{L.R}$ in eqn (3)):

$$
\begin{align}
k(t) &= \frac{t^2 - 5t - 2}{12(t-1)^3} + \frac{t\ln t}{2(t-1)^4} \tag{45} \\
&\rightarrow \frac{1}{6} + \frac{t\ln t}{2} \quad , \quad t \rightarrow 0 \\
&\rightarrow \frac{1}{12t} \quad , \quad t \rightarrow \infty \\
\overline{k}(t) &= \frac{2t^2 + 5t - 1}{12(t-1)^3} - \frac{t^2 \ln t}{2(t-1)^4} \tag{46} \\
&\rightarrow \frac{1}{12} \quad , \quad t \rightarrow 0 \\
&\rightarrow \frac{1}{6t} \quad , \quad t \rightarrow \infty \\
k_f(t) &= \frac{t-3}{2(t-1)^2} + \frac{\ln t}{(t-1)^3} \tag{47} \\
&\rightarrow -\frac{3}{2} - \ln t \quad , \quad t \rightarrow 0 \\
&\rightarrow \frac{1}{2t} \quad , \quad t \rightarrow \infty \\
\overline{k_f}(t) &= \frac{t+1}{2(t-1)^2} - \frac{t\ln t}{(t-1)^3} \tag{48} \\
&\rightarrow \frac{1}{2} + t\ln t \quad , \quad t \rightarrow 0 \\
&\rightarrow \frac{1}{2t} \quad , \quad t \rightarrow \infty \\
\overline{y}(t) &= \frac{-4t^3 + 45t^2 - 33t + 10}{12(t-1)^3} - \frac{3t^3 \ln t}{2(t-1)^4} \tag{49} \\
&\rightarrow -\frac{1}{3}\left(1 - \frac{33}{4t} + \frac{9\ln t}{2t}...\right) \quad t \rightarrow \infty \\
&\rightarrow -\frac{5}{6}\left(1 - \frac{3t}{10} + ...\right) \quad t \rightarrow 0
\end{align}
$$

where $t = m_f^2/m_B^2$, and m_B is the boson mass.

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